QM-2 September 9

Today: Linear stark effect
(5.2.1)

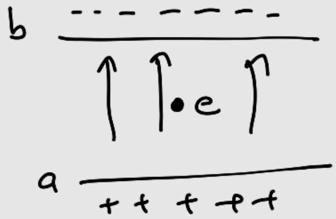
Warm-up

Potential energy of electron (charge $-e$)
in $\vec{E} = \mathcal{E}\hat{e}_z$

$\Rightarrow$

$$\vec{E} = \mathcal{E}\hat{e}_z$$

$$-\frac{\partial V}{\partial z} = \mathcal{E}\hat{e}_z$$



$$\Rightarrow V = -(b-a)\mathcal{E}$$

$\hookrightarrow$ potential.

potential energy $= qV = -e \cdot V$

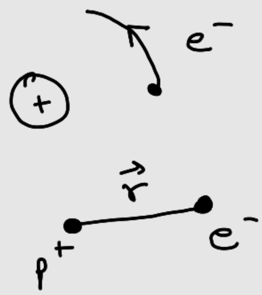
$$\boxed{U = (b-a)e\mathcal{E}}$$

$$\boxed{U = e\mathcal{E}z}$$

Linear Stark effect

apply $\vec{E} = \mathcal{E}\hat{z}$ to hydrogen atom

$\Rightarrow V = e\mathcal{E}z$



Find 1^{st} order energy shifts

a) ground state ($n = 1$)

b) first excited ($n=2$)

c) 2^{nd} excited ($n=3$) $\rightarrow$ H.W.

$\Rightarrow$ a) Ground state (non-degenerate):

$$|nlm\rangle = |n=1, l=0, m=0\rangle$$

$$H_0|nlm\rangle = E_n^{(0)}|nlm\rangle$$

since there is only one energy state (for $n=1$)

$$H_0|n=1\rangle = E_1^{(0)}|n=1\rangle$$

$$\Rightarrow \Delta^{(1)}_{n=1} = \langle n=1|V|n=1\rangle$$

$$= \int_0^\infty dr \underbrace{\int d\Omega}_{\int_0^\pi d\theta \int_0^{2\pi} d\phi \sin\theta} r^2 \, |\psi_{100}(\vec{r})|^2 \, \mathcal{E} \underbrace{z}_{\text{odd}}$$

$\Rightarrow$ entire f^n is odd.

$$\boxed{\Delta^{(1)}_{n=1} = 0}$$

$\Rightarrow$ no first order stark shift to the ground state

- Find 1^{st} order energy shift for $n=2$, unperturbed states (degenerate)

n	l	m
2	0	0
2	1	0
2	1	1
2	1	-1

"4-fold degenerate"

$|m_1^{(0)}\rangle = |2\,0\,0\rangle$

$|m_2^{(0)}\rangle = |2\,1\,0\rangle$

$|m_3^{(0)}\rangle = |2\,1\,1\rangle$

$|m_4^{(0)}\rangle = |2\,1\,-1\rangle$

Degenerate Pert. theory

$$H = H_0 + \lambda V$$

Solve: $H|l\rangle = E_l|l\rangle$

$$|l\rangle = |l^{(0)}\rangle + \lambda|l^{(1)}\rangle + \ldots\ldots$$

$\lambda \to 0$: $|l\rangle \to |l^{(0)}\rangle = ?$

Find $|l^{(0)}\rangle$

know: $H_0|m_i^{(0)}\rangle = E_D^{(0)}|m_i^{(0)}\rangle$, $i = 1$ to g

Solve:

$$\begin{pmatrix} \langle m_1^{(0)}|V|m_1^{(0)}\rangle & \cdots & \langle m_1^{(0)}|V|m_g^{(0)}\rangle \\ \vdots & \ddots & \vdots \\ \langle m_g^{(0)}|V|m_1^{(0)}\rangle & \cdots & \langle m_g^{(0)}|V|m_g^{(0)}\rangle \end{pmatrix}$$

$\Rightarrow$ eigen values : $\Delta_i^{(1)}$

eigenvecs : $\begin{pmatrix} a_{i1} \\ \vdots \\ \vdots \\ a_{ig} \end{pmatrix}$

Selection Rules

- need $\langle n'l'm'|V|n\,l\,m\rangle$ for $V \propto z$
- often zero
- Parity selection Rule : $\psi_{nlm}(\vec{r}) = (-1)^l \psi_{nlm}(-\vec{r})$

$$\langle n'l'm'|V|nlm\rangle = \int d^3r \underbrace{\psi^*_{n'l'm'}(\vec{r})\,\psi_{nlm}(\vec{r})}_{\text{can be even or odd depending on } l} \underbrace{z}_{\text{odd}} e\,\mathcal{E}$$

$= 0$, if l, l' same parity

m Selection Rule

$\psi_{nlm} \propto Y_l^m \propto e^{im\phi}$

$$\langle n'l'm'|V|nlm\rangle \propto \int_0^{2\pi} d\phi\, e^{i(m-m')\phi}$$

$$= \boxed{0 \quad \text{if } m \neq m'}$$

$\rightarrow$ for $n=2$, which elements can be non-zero?

(l, l' opposite parity, $m=m'$)

$\rightarrow$ only $|m_1^{(0)}\rangle$ & $|m_2^{(0)}\rangle$

$|200\rangle \quad |210\rangle$

$$\therefore \langle 200|V|210\rangle = \int d^3r \, \psi_{200}^*(\vec{r}) \, e\mathcal{E}z \, \psi_{210}(\vec{r})$$

from Sakurai appendix or any other integral table.

$$= -3e\mathcal{E}a_0$$

matrix on $n=2$, subspace:

$$\begin{array}{cc} [V]_{n=2} = -3e\mathcal{E}a_0 \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{ccc} n & l & m \\ 2 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & -1 \end{array} \end{array}$$

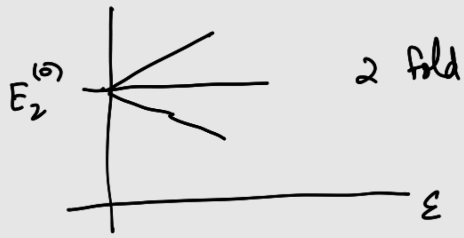
eigenvalues : $\Delta = (-3e\mathcal{E}a_0)\mu$

$$0 = \begin{vmatrix} -\mu & 1 & 0 & 0 \\ 1 & -\mu & 0 & 0 \\ 0 & 0 & -\mu & 0 \\ 0 & 0 & 0 & -\mu \end{vmatrix}$$

$$= \mu^2 \begin{vmatrix} -\mu & 1 \\ 1 & -\mu \end{vmatrix}$$

$$= \mu^2(\mu^2 - 1)$$

$\Rightarrow$ solutions: $\mu = 0, 0, -1, +1$



2 fold-degenerate