QM-2 Sep-23

Warm-up: Hydrogen (including spin)

How many states with $n=2$?

a) $|n=2, l, m_l m_s\rangle$ (uncoupled)

b) $|n=2, l, j, m_j\rangle$ (coupled)

a)

$\Rightarrow$ $n=2$, $l = 0, 1$, $m_l = 0$

$= -1, 0, 1$

$m_s = -1/2, 1/2$

# of states $= 4 \times 2 = \boxed{8 \text{ states}}$

b)

$s = 1/2$ $\quad J = L + S$

l	j	m_j
0	1/2	$\pm 1/2$
1	1/2	$\pm 1/2$
1	3/2	$\pm 1/2, \pm 3/2$

$\Rightarrow \boxed{8 \text{ states.}}$

* Revise addition of angular momentum: just what the j & s variables mean in coupled basis and what values they can take in coupled basis.

* Do problems on angular momentum addition from lim series.

Zeeman Effect in Hydrogen

- apply $\vec{B} = B\hat{e}_z$

$$V_B = -\vec{\mu}\cdot\vec{B}$$

$$\vec{\mu} = -\frac{e}{2m_e}\vec{L} - \frac{e}{m_e}\vec{S} \qquad (e>0)$$

$$\Rightarrow \boxed{V_B = \frac{eB}{2m_e}(L_z + 2S_z)}$$

$\vec{J} = \vec{L} + \vec{S}$

$$\boxed{[V_B, J_z] = \frac{eB}{2m_e}[L_z + 2S_z, L_z + S_z] = 0}$$

$\Rightarrow$ m_j is conserved

$$[V_B, \vec{L}^2] = 0 = [V_B, \vec{S}^2]$$

(from results of momentum alge

$$[V_B, L_z] = 0 = [V_B, L_z]$$

$\Rightarrow$ V_B is diagonalized in uncoupled basis $|n\, l\, m_l\, m_s$

but : $[V_B, \vec{J}^2] \neq 0$

- with Relativistic corrections :

$$V = \underbrace{V_{KE}}_{\text{both}} + V_D + \underbrace{V_{LS}}_{\substack{\text{coupled} \\ (|n\,l\,j\,m_j\rangle)}} + \underbrace{V_B}_{\substack{\text{uncoupled} \\ (|n\,l\,m_l\,m_s\rangle)}}$$

diagonal in:

$[V, J_z] = 0 \rightarrow m_j$ conserved

$$\begin{pmatrix} \boxed{m_j} & & & \\ & \boxed{m_j} & & \\ & & \boxed{m_j} & \\ & & & \ddots \end{pmatrix}$$

* Exercise

Neglect relativistic corrections

- Find $\Delta^{(1)}$ due to V_B
- V_B is diag. in $|n\,l\,m_l\,m_s\rangle$ basis.

$\Rightarrow$ inner product $\langle n\,l'\,m_l'\,m_s' | V_B | n\,l\,m_l\,m_s\rangle$

$$[V_B]_{n=2} = \begin{pmatrix} \langle n=2, l=0, m_l=0, m_s=\tfrac{1}{2} | V_B | \text{''} \rangle & 0 & \cdots & \\ 0 & \langle \ldots \rangle & & \\ & & \langle \ldots \rangle & \\ & & & \ddots \end{pmatrix}$$

$$\Delta^{(1)}_{n l m_s m_s} = \langle n\,l\,m_s\,m_s | V_B | n\,l\,m_l\,m_s \rangle$$

$$= \langle n\, l\, m_l\, m_s | \frac{eB}{2m_e} (\underbrace{L_z}_{\hbar m_l} + 2\underbrace{S_z}_{\hbar m_s}) | n\, l\, m_l m_s \rangle$$

1st order energy shift without relativistic effects

$$\Delta^{(1)} = \left(\frac{e\hbar}{2m_e}\right) B \cdot (m_l + 2m_s) = \underbrace{\mu_B}_{\text{Bohr magneton: } 9.27\times 10^{-24}\ J/T} B\, (m_l + 2m_s)$$

Consider $B = 1\,T$

$$\Rightarrow \Delta^{(1)} \sim \mu_B B \sim \boxed{10^{-23}\,J}$$

compare to relativistic

$$\Delta^{(1)}_{REL} \sim \alpha^4 m_e c^2 \sim \boxed{10^{-22}\,J}$$

Zeeman + Relativistic effects

- $V = V_{Rel} + V_B$
- conserved : $[V_B, J_z] = 0 \rightarrow m_j$

  $[V_B, \vec{L}_z^2] = 0 \rightarrow l$

  $\rightarrow$ coupled basis $|n l_j m_j\rangle$

Find eigenvalues / vectors of matrices :

$$[V_{nlm_j}]_{j,j'} = \langle n\, l\, j'\, m_j | V | n\, l_j\, m_j \rangle$$

blocks

* would be useful on homework problem.

- for $l=0$: $j=\frac{1}{2} \rightarrow 1\times1$ matrix
- for $l>0$: $j=l\pm\frac{1}{2}$, $|m_j|\leq j$
  - If $|m_j|=l+\frac{1}{2}$, then $j=l+\frac{1}{2}$

    in that case $\rightarrow 1\times1$ matrix
  - If $|m_j|<l+\frac{1}{2}$ then $j=l\pm\frac{1}{2}$

    $\rightarrow 2\times2$ matrix

Ex:

$m_j=3/2 \rightarrow j=3/2 \rightarrow 1\times1$

$m_j=1/2 \rightarrow j=1/2, 3/2 \rightarrow 2\times2$

$l=0$	1×1	$1+1+4+2=8$
$l=1$	2	2×2
	2	1×1