QM 2

Oct 23

Today:

Interaction with EM field

$(\sim 5.8.1 - 2)$

- electric dipole interaction
- selection rules

WARM UP

$$\vec{E} = E_1 \left(\cos(\omega t), \sin(\omega t), 0 \right)$$

$$\equiv E_0 \hat{\varepsilon} e^{-i\omega t} + E_0 \hat{\varepsilon}^* e^{i\omega t}$$

Let $\hat{\varepsilon}^* \cdot \hat{\varepsilon} = 1$, $E_0 \in \mathbb{R}$

Find $\hat{\varepsilon}$, E_0

$\Rightarrow$ starting out with writing trig f^n in LHS in e.

$$E_1 \left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}, \frac{e^{i\omega t} - e^{-i\omega t}}{2i}, 0 \right)$$

$$\equiv E_0 \hat{\varepsilon} e^{-i\omega t} + E_0 \hat{\varepsilon}^* e^{i\omega t}$$

$$\Rightarrow \frac{E_1}{2i}\left[i\left(e^{i\omega t}+e^{-i\omega t}\right)\hat{x}+\left(e^{i\omega t}-e^{-i\omega t}\right)\hat{y}\right.$$

$$\left.+\ 0\hat{k}\right] = (\quad)$$

$$\Rightarrow \frac{E_1}{2i}\left[e^{i\omega t}\left(i\hat{x}+\hat{y}\right)+e^{-i\omega t}\left(i\hat{x}-\hat{y}\right)\right.$$

$$\left.\right] = (\quad)$$

comparing coefficients of $e^{i\omega t}$ & $e^{-i\omega t}$ on both sides:

$$\therefore \quad \frac{E_1}{2i}\left(i\hat{x}+\hat{y}\right) = E_0\hat{\varepsilon}$$

and, $$\frac{E_1}{2i}\left(i\hat{x}-\hat{y}\right) = E_0\hat{\varepsilon}^*$$

dotting these equations to use the given condition $\varepsilon^*\cdot\varepsilon = 1$

$$\Rightarrow -\frac{E_1^2}{4}\left(-1-1\right) = E_0^2$$

$$\Rightarrow \boxed{E_0 = \pm \frac{E_1}{\sqrt{2}}}$$

So $$\frac{E_1}{2i}(i\hat{x}+\hat{y}) = \pm \frac{E_1}{\sqrt{2}}\hat{\varepsilon}$$

$$\Rightarrow \begin{cases} \hat{\varepsilon} = \mp i\sqrt{2}\,(i\hat{x}+\hat{y}) \\ \hat{\varepsilon}^* = \pm i\sqrt{2}\,(-i\hat{x}+\hat{y}) \end{cases}$$

normalising

$$\begin{cases} \hat{\varepsilon} = \mp i\left(\frac{i\hat{x}+\hat{y}}{\sqrt{2}}\right) \\ \hat{\varepsilon}^* = \pm i\left(\frac{-i\hat{x}+\hat{y}}{\sqrt{2}}\right) \end{cases}$$

— x —

- $\hat{\varepsilon} = \frac{1}{\sqrt{2}}(\hat{e}_x + i\hat{e}_y)$, $\hat{\varepsilon}^* = \frac{1}{\sqrt{2}}(\hat{e}_x - i\hat{e}_y)$

these can be use as basis vectors for writing electric field

* spherical basis w.r.t z axis
* useful for calculating angular momentum.

Spherical basis

$$\hat{e}_1 = \frac{-1}{\sqrt{2}}(\hat{e}_x + i\hat{e}_y)$$

(minus sign would be convenient in calculations later)

$$\hat{e}_0 = \hat{e}_z$$

$$\hat{e}_{-1} = \frac{1}{\sqrt{2}}(\hat{e}_x - i\hat{e}_y)$$

$$\hat{e}_1^* = \frac{-1}{\sqrt{2}}(\hat{e}_x - i\hat{e}_y)$$

$$= -\hat{e}_{-1}$$

$$e_0^* = e_0$$

can be summarised with identity:

$$\boxed{\hat{e}_q^* = (-1)^q \hat{e}_{-q}}, \text{ with } q = 0, \pm 1$$

Electric dipole interaction (approximation)

- the interaction can be defined with some time-depend-
  - ent perturbation:

$$V(t) \approx -q\,\vec{r}\cdot\vec{E}\,(\vec{r}=0,t)$$

- bound state, $\vec{r} \approx 0$
- valid when $\lambda >>$ system size.
  - for atoms $\lambda \sim 10^{-7}\,m$, $r \sim 10^{-10}\,m$

(long wavelength approximation)

* this is an approximation: we are looking at the electric field at only one point.

- dipole mom.: $\vec{d} = q\,\vec{r}$

$$\vec{d} = \sum_j q_j\,\vec{r}_j \rightarrow V = -\vec{d}\cdot\vec{E}\,(0,t)$$

- apply perturbation:
  - arbitrary polarization

$$\vec{E}(0,t) = E_0\,\hat{\varepsilon}\,e^{-i\omega t} + E_0^*\,\hat{\varepsilon}^*\,e^{i\omega t}$$

- perturbation :

$$\begin{aligned} V(t) &= -\vec{d}\cdot\vec{E}(0,t) = e\vec{r}\cdot\vec{E}(0,t) \\ &= e E_0 (\vec{r}\cdot\hat{\varepsilon})\,e^{-i\omega t} + e E_0^*(\vec{r}\cdot\hat{\varepsilon}^*)\,e^{i\omega t} \\ &= \quad v^{+} e^{-i\omega t} \quad + \quad v\, e^{i\omega t} \end{aligned}$$

where,
$$v = e E_0^* (\vec{r}\cdot\hat{\varepsilon}^*)$$
$$v^{+} = e E_0 (\vec{r}\cdot\hat{\varepsilon})$$

consider , $\hat{\varepsilon} = \hat{e}_q$

→ possible transitions : – bound state → bound state

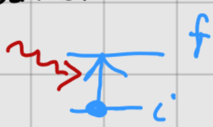
– $|i\rangle = |n_i\, l_i\, m_i\rangle$
$\downarrow$
$|f\rangle = |n_f\, l_f\, m_f\rangle$

(non-relativistic Hydrogen atom)

<u>Transition Matrix Element</u>: $\langle f|V(t)|i\rangle$

- in harmonic pert. th. , it was derived that :

absorption : $\langle f|v^{+}|i\rangle$



emission : $\langle f|v|i\rangle$

<u>absorption :</u>

$$\begin{aligned} \langle f|v^{+}|i\rangle &= \langle f|e E_0\, \vec{r}\cdot\hat{e}_q|i\rangle \\ &= e E_0 \underbrace{\langle f|\vec{r}\cdot\hat{e}_q|i\rangle} \end{aligned}$$

called the dipole matrix element.

$$\bullet\ \vec{r}\cdot\hat{e}_q = r_q = \begin{cases} -\frac{1}{\sqrt{2}}(x+iy), & q=1 \\ z, & q=0 \\ \frac{1}{\sqrt{2}}(x-iy), & q=-1 \end{cases}$$

↳ components of $\vec{r}$ in the spherical basis defined in WARM-UP.

minus sign for $q=1$ helps write kets in terms of spherical harmonics.

$$= \sqrt{\frac{4\pi}{3}}\, Y_1^q(\theta,\phi) \ \ldots\ldots$$

$$\langle f|\vec{r}\cdot\hat{e}_q|i\rangle = \underbrace{\int_0^\infty dr\, r^2 r\, R_f(r) R_i(r)}_{\neq 0} \underbrace{\int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\phi\, Y_{l_f}^{m}(\theta,\phi)^*\, Y_1^q(\theta,\phi)\, Y_{l_i}^{m_i}(\theta,\phi) \sqrt{\frac{4\pi}{3}}}_{\equiv I}$$

$\equiv I$ Sakurai (3.393)

$$I \propto \langle l_f m_f | 1 q, l_i m_i \rangle$$

$= 0$, unless: $m_f = m_i + q$

and $l_f = |l_i - 1|,\ \cancel{l_i}\ 0,\ l_i + 1$

and l_f & l_i have opposite parity

$$\boxed{l_f = l_f \pm 1}$$

Similarly for

Emission:

$$\langle f | \vec{r} \cdot \hat{e}_q^* | i \rangle = (-1)^q \langle f | \vec{r} \cdot \hat{e}_{-q}^* | i \rangle$$

$= 0$ unless:

$$\begin{cases} m_f = m_i - q \\ l_f = l_i \pm 1 \end{cases}$$

- For a general polarisation

$$\left.\boxed{\begin{matrix} m_f = m_i + q \;, \; q = 0, \pm 1 \\ l_f = l_i \pm 1 \end{matrix}}\right\} \Delta n = \text{any}$$

Example: which are allowed:

a) $|1\;0\;0\rangle \longrightarrow |2\;0\;0\rangle$

b) $|1\;0\;0\rangle \longrightarrow |2\;1\;0\rangle$

c) $|1\;0\;0\rangle \longrightarrow |3\;2\;1\rangle$

$\Rightarrow$ a) not allowed (l cond^n won't satisfy)

b) allowed

c) not allowed.