Nov-4

Today: Scattering (6.1, 6.2)

WARM UP: Scattering from $V(\vec{r})$

$$\frac{d\sigma}{d\Omega} \approx \left(\frac{mL^3}{2\pi\hbar^2}\right)^2 |V_{fi}|^2$$

$$\langle \vec{r} | \vec{k}_f \rangle = \frac{1}{L^{3/2}} e^{i\vec{k}_f \cdot \vec{r}}$$

$$\langle \vec{r} | \vec{k}_i \rangle = \frac{1}{L^{3/2}} e^{i\vec{k}_i \cdot \vec{r}}$$

a) write $\frac{d\sigma}{d\Omega}$ as integral on $V(\vec{r})$

b) for $V(\vec{r}) = g\,\delta(\vec{r})$, find $\frac{d\sigma}{d\Omega}$

⇒ a) $$|V_{fi}|^2 = \left|\int \left(\frac{1}{L^{3/2}} e^{-i\vec{k}_f \cdot \vec{r}}\right) V \left(\frac{1}{L^{3/2}} e^{+i\vec{k}_i \cdot \vec{r}}\right) d^3\vec{r}\right|^2$$

⋮

$\vdots$

$$\frac{d\sigma}{d\Omega} = \left(\frac{m l^3}{2\pi\hbar^2}\right)^2 \left(\frac{1}{l^3}\right)^2 \left| \int e^{-i\vec{k}_f\cdot\vec{r}} N e^{i\vec{k}_i\cdot\vec{r}} \, d^3\vec{r} \right|^2$$

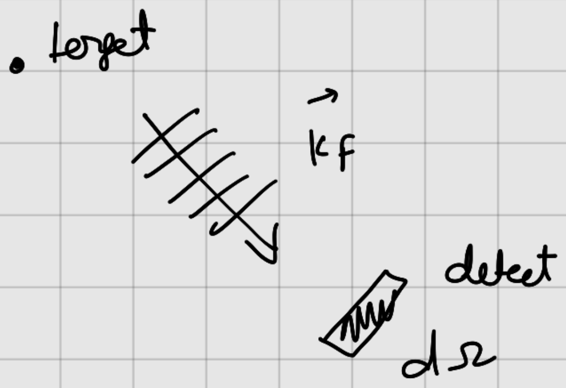
(b)

$\vdots$

$$\frac{d\sigma}{d\Omega} = \left(\frac{m}{2\pi\hbar^2}\right)^2 \left| g e^0 \right|^2$$

$$= \left(\frac{m|g|}{2\pi\hbar^2}\right)^2$$

Quantum Wave Scattering:



$$\text{flux} = \frac{\text{prob.}}{\text{area} \times \text{time}} = \frac{1}{L^2 (L/v)} = \frac{\hbar k_i / m}{L^3}$$

then,

$$\text{detection rate} = \text{flux} \left(\frac{d\sigma}{d\Omega} \right) d\Omega$$

(from T.D.P.T)

- Outline (2 types of problems)
  1) Born approximation.
  2) Partial wave expansion / scattering.
     formal Scattering theory

Zettili develops scattering theory using time independent Schrö. eqn instead of T.D.P.T, like Sakurai.

* Derive from T.D.P.T (Sakurai)

* Transition Rate from $\vec{k}_i \to \vec{k}_f$

- $V(\vec{r})$ turned on slowly

$$\tilde{V}(\vec{r}, t) = \underbrace{e^{\varepsilon t} V(\vec{r})}, \quad \varepsilon > 0$$

↳ potential (due to exponential term) is being turned on slowly from Particle's perspective.

Time evolution operator:

$$\langle n | U_I(t, -\infty) | i \rangle = \delta_{ni} + \frac{1}{i\hbar} \int_{-\infty}^{\infty} dt' \langle n | V_I(t') U_I(t', -\infty) | i \rangle e^{\varepsilon t'}$$

(in the limit as $\varepsilon \to 0^+$)

$$= \delta_{ni} + \frac{1}{i\hbar} \underbrace{T_{ni}}_{\text{some unknown (for now)}} \int_{-\infty}^{t} dt' \, e^{(i\omega_{ni}+\varepsilon)t'}$$

MOTIVATION: First order

$$\langle n| U_I(t,-\infty)|i\rangle = \delta_{ni} + \frac{1}{i\hbar}\int_{-\infty}^{t} dt' \langle n|V_I(t')|i\rangle e^{\varepsilon t'}$$

$$+ \delta_{ni} + \frac{1}{i\hbar}\int_{-\infty}^{t} dt' \langle n| e^{iH_0 t'/\hbar} V e^{-iH_0 t'/\hbar} |i\rangle e^{\varepsilon t'}$$

$$= \delta_{ni} + \frac{1}{i\hbar}\int_{-\infty}^{t} dt' \, e^{i(\omega_{ni}+\varepsilon)t'} \underbrace{V_{ni}}_{T_{ni}}$$

Similarly, 2nd order:

$$\langle n|U_I(t,-\infty)|i\rangle = \delta_{ni} + \frac{1}{i\hbar}\underbrace{\left(V_{ni} + \sum_m \frac{V_{nm}V_{mi}}{E_i - E_m + i\hbar\varepsilon}\right)}_{T_{ni}} \int_{-\infty}^{t} dt' \, e^{i(\omega_{ni}+\varepsilon)t'}$$

T-Matrix

$$T(E) = V + V \frac{1}{E - H_0 + i\hbar\varepsilon} T(E)$$

T-Matrix: $T_{ni}(E = E_i = E_n) = V_{ni} + \sum_m \frac{V_{nm} T_{mi}}{E_i - E_m + i\hbar\varepsilon}$

Scattering Cross Section

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{mL^3}{2\pi\hbar^2}\right) |T_{fi}|^2}$$ (exact)

—x—

Lippmann-Schwinger Equation

- Define "scattering state" $|\psi(t)\rangle$

$$T|i\rangle = V|\psi^{(+)}\rangle$$

Equation for $|\psi^{(+)}\rangle$

$$T = V + VG_0T$$

$$G_0 = \frac{1}{E - H_0 + i\eta}$$

$$T|i\rangle = V|i\rangle + VG_0T|i\rangle$$

$$V|\psi^{(+)}\rangle = V|i\rangle + VG_0V|\psi^{(+)}\rangle$$

$$|\psi^{(+)}\rangle = |i\rangle + G_0(E)V|\psi^{(+)}\rangle$$

Lippmann Schwinger eqn

$$\boxed{|\psi^{(+)}\rangle = |i\rangle + \frac{1}{E-H_0}V|\psi^{(+)}\rangle} \quad \eta \to 0^+$$

<u>$|\psi^{(+)}\rangle$ solves the T.I.S.E:</u>

$$(E-H_0+i\eta)|\psi^{(+)}\rangle = \underbrace{(\underbrace{E-H_0}_{E_i-E_i=0}+\underbrace{i\eta}_{\lim \eta\to 0^+})|i\rangle}_{=0} + V|\psi^{(+)}\rangle$$

$$\therefore \quad (E-H_0)|\psi^{(+)}\rangle = V|\psi^{(+)}\rangle$$

$$\boxed{E|\psi^{(+)}\rangle = (H_0+V)|\psi^{(+)}\rangle}$$

L.S.E in Integral form

$$\psi^{(+)}(\vec{r}) = \langle\vec{r}|\psi^{(+)}\rangle = \underbrace{\langle\vec{r}|i\rangle}_{\phi_i(\vec{r})} +$$

$$\langle \vec{r} | \frac{1}{E_i - H_0 + i\eta} V | \psi^{(+)} \rangle$$

insert identity operator.

$$\mathbb{1} = \int d^3\vec{r}\,' |\vec{r}\,'\rangle\langle\vec{r}\,'|$$

$$= \phi_i(\vec{r}) + \int d^3\vec{r}\,' \langle \vec{r} | \frac{1}{E_i - H_0 + i\eta} | \vec{r}\,' \rangle V(\vec{r}\,') \psi^{(+)}(\vec{r}\,')$$

Green's function :

Define : $G_+(\vec{r}, \vec{r}\,') \equiv \frac{\hbar^2}{2m} \langle \vec{r} | \frac{1}{E_i - H_0 + i\eta} | \vec{r}\,' \rangle$

$$\Rightarrow \boxed{\psi^{(+)}(\vec{r}) = \phi_i(\vec{r}) + \frac{2m}{\hbar^2} \int d^3\vec{r}\,' G_+(\vec{r}, \vec{r}\,') V(r') \psi^{(+)}(\vec{r}\,')}$$

Lsintegral L.S.E