Today : Density operators (Sakurai 6.4)

↳ "won't be on exam, but it's interesting"

– Dr. Sommer

WARM-UP: Spin $\frac{1}{2}$

$$|\alpha\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$

Measure S_x

Find possible results & probabilities.

my answer – – – – – – – – – – – – – – – –

⇒ if spin is along $\pm S_x$

$$\langle\alpha|S_x\rangle = \frac{1}{\sqrt{2}}$$, result would be $\pm\frac{\hbar}{2}$

probability of $S_x = \langle\alpha|S_x\rangle^2 = \frac{1}{2}$

if spin is along z or y.

$$\langle\alpha|S_x\rangle = \frac{1}{\sqrt{2}}(\langle\uparrow| + \langle\downarrow|)\left(\frac{1}{\sqrt{2}}(|\uparrow\rangle \pm i|\downarrow\rangle)\right)$$

$$= \frac{1}{2}(1 \pm i)$$

probability of $S_x = |\langle\alpha|S_x\rangle|^2 = 1$

result would be eigenvalue of x.

\# - - - - - - - - - - - - (wrong) - #

\# Class answers - - - - - - - - - - - - - - #

$$S_x = \frac{\hbar}{2}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$$S_x|a\rangle = \frac{\hbar}{2}\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}\frac{1}{\sqrt{2}}\left\{\begin{bmatrix}0\\1\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}\right\} = \frac{\hbar}{2\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$$

$$= \frac{\hbar}{2}\left[\frac{1}{\sqrt{2}}|\downarrow\rangle + \frac{1}{\sqrt{2}}|\uparrow\rangle\right]$$

$$= \frac{\hbar}{2}|a\rangle$$

$\Rightarrow$ 100% probability of $S_x = \frac{\hbar}{2}$

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\# Q&A - - - - - - #

1) How much degenerate Pert. th. on exam?

$\Rightarrow$ Summary of topics: (Dr. Sommer's)

- 0^{th} order eigenstates.

• first order energy shifts.

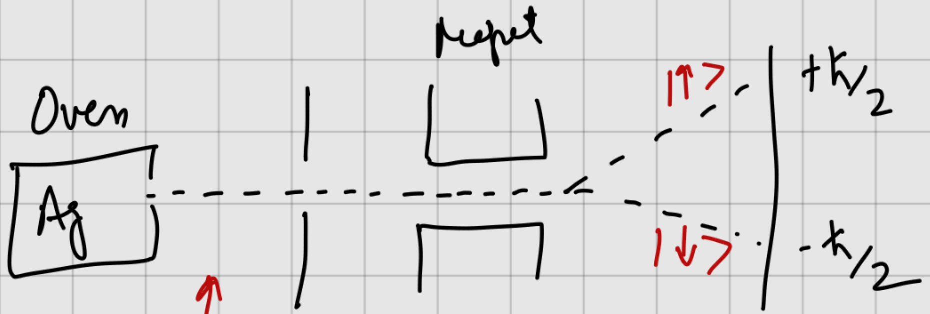
diagonalize.

$$[V]_D = \begin{pmatrix} \langle \varphi_1 | V | \varphi_1 \rangle & \cdots & \langle \varphi_1 | V | \varphi_n \rangle \\ \vdots & \ddots & \vdots \\ \langle \varphi_n | V | \varphi_1 \rangle & \cdots & \langle \varphi_n | V | \varphi_n \rangle \end{pmatrix}$$

$$\Delta E^{(1)} = \lambda \quad ; \quad |n\rangle = \sum_m c_{nm} |\varphi_m\rangle$$

Density operators

Stern-Gerlach :



$|\psi\rangle = ?$

- maybe the wavef^n is $|\alpha\rangle = \frac{1}{\sqrt{2}} |\uparrow\rangle + \frac{1}{\sqrt{2}} |\downarrow\rangle$

but, if we measure S_x, then.

$$P(S_x = \hbar/2) = 1/2$$

$$P(S_x = -\hbar/2) = 1/2$$

but using $|\alpha\rangle$, we get.

$$P(S_x = \hbar/2) = 1$$

So $|\alpha\rangle$ is not what we are looking for.
and this problem happens for every ket that we write.

for any $|\alpha\rangle = \cos\frac{\theta}{2}\,|\uparrow\rangle + e^{i\phi}\sin\frac{\theta}{2}\,|\downarrow\rangle$

eig. of $\vec{S}\cdot\hat{n}$, $\hat{n} = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$

→ Density Matrices

- observables $A = A^{\dagger}$
- knowing "state" means we can find $\langle A\rangle$

$$\langle A\rangle = \sum_{ij} \rho_{ji}\,\langle i|A|j\rangle$$

$$= \sum_{ij} \langle j|\rho\underbrace{|i\rangle\langle i|}_{\mathbb{1}}A|j\rangle$$

$$= \sum_{ij} \langle j|\rho A|j\rangle = \mathrm{tr}(\rho A)$$

Properties of ρ

Given $\langle A \rangle = \mathrm{tr}(\rho A)$

1. $\mathrm{tr}(\rho) = 1$ $\quad (A = \mathbb{I})$
2. $\rho = \rho^{+}$ $\quad (\langle A \rangle \in \mathbb{R})$
3. eig. vals of ρ are ≥ 0

Pure states

- if we have state vec $|\alpha\rangle$

$$\rho = |\alpha\rangle\langle\alpha|$$

$$\mathrm{tr}(\rho A) = \sum_j \langle j|\rho A|j\rangle$$

$$= \sum_j \langle j|\alpha\rangle\langle\alpha|A|j\rangle$$

$$= \sum \langle\alpha|A\underbrace{|j\rangle\langle j|}_{1}\alpha\rangle$$

$$= \langle\alpha|A|\alpha\rangle$$
$$= \langle A \rangle$$

Mixed states.

e.g. 50% $|\uparrow\rangle$

50% $|\downarrow\rangle$

$$\rho = \frac{1}{2}\,|\uparrow\rangle\langle\uparrow| + \frac{1}{2}\,|\downarrow\rangle\langle\downarrow|$$

$$= \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

$$\mathrm{tr}(\rho) = 1/2 + 1/2 = 1$$